On Some Transformation Formulas for The $\overline{H}$-Function

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Abstract: In the present paper we establish four transformations of double infinite series involving the $\overline{H}$ -function. These formulas are then used to obtain double summation formulas for the said function. Our results are quite general in character and a number of summation formulas can be deduced as particular cases. Several interesting special cases of our main finding have been mentioned briefly.

Key words: $\overline{H}$ -function, Gauss's summation theorem, Double infinite series
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1. Introduction

The $\overline{H}$ -function occurring in the paper will be defined and represented as follows:

$$\overline{H}^{M,N}_{P,Q}\left[z\right]=\overline{H}^{M,N}_{P,Q}\left[z\left|\begin{matrix}(a_j;\alpha_j;A_j)_{1,N},(a_j;\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\overline{\phi}(\xi)z^{\xi}d\xi \tag{1.1}$$

where
$$\overline{\phi}(\xi)=\frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\left\{\Gamma(1-a_j+\alpha_j\xi)\right\}^{A_j}}{\prod_{j=M+1}^{Q}\left\{\Gamma(1-b_j+\beta_j\xi)\right\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)} \tag{1.2}$$

Which contains fractional powers of the gamma functions. Here, and throughout the paper $a_j(j=1,...,p)$ and $b_j(j=1,...,Q)$ are complex parameters, $\alpha_j\geq 0(j=1,...,P),\beta_j\geq 0(j=1,...,Q)$ (not all zero simultaneously) and exponents $A_j(j=1,...,N)$ and $B_j(j=N+1,...,Q)$ can take on non integer values.

The following sufficient condition for the absolute convergence of the defining integral for the $\overline{H}$ -function given by equation (1.1) have been given by Buschman and Srivastava [1].

$$\Omega\equiv\sum_{j=1}^{M}\left|\beta_j\right|+\sum_{j=1}^{N}\left|A_j\alpha_j\right|-\sum_{j=M+1}^{Q}\left|\beta_jB_j\right|-\sum_{j=N+1}^{P}\left|\alpha_j\right|>0 \tag{1.3}$$

and $\left|\arg(z)\right|<\frac{1}{2}\pi\Omega$ (1.4)

The behavior of the $\overline{H}$ -function for small values of $|z|$ follows easily from a result recently given by Rathie ([7],p.306,eq.(6.9)).
We have

$$\overline{H}^{M,N}_{P,Q}\left[z\right]=0\left(|z|^{\gamma}\right),\gamma=\min_{1\leq j\leq N}\left[\operatorname{Re}\left(b_j/\beta_j\right)\right],|z|\to 0 \tag{1.5}$$

If we take $A_j=1(j=1,...,N)$, $B_j=1(j=M+1,...,Q)$ in (1.1), the function $\overline{H}^{M,N}_{P,Q}$ reduces to the Fox's H-function [3].
We shall use the following notation:

$A^{*}=\left(a_{j},\alpha_{j};A_{j}\right)_{1,N},\left(a_{j},\alpha_{j}\right)_{N+1,P}$ and $B^{*}=\left(b_{j},\beta_{j}\right)_{1,M},\left(b_{j},\beta_{j};B_{j}\right)_{M+1,Q}$

2. Transformation Formulas:

In this section we establish the following four transformation Formulas for the $\overline{H}$ -function:

First formula

$$\sum_{m,n=0}^{\infty} x^{m} y^{n} \overline{H}_{P+2,Q+1}^{M,N+2}\left[z\left|\begin{matrix}(1-a-m,\rho;1),(1-b-n,\sigma;1),A^{*}\\ B^{*},(1-a-b-m-n,\sigma+\rho;1)\end{matrix}\right.\right]=(x+y-xy)^{-1}$$

$$\sum_{s=0}^{\infty} x^{s+1} \overline{H}_{P+2,Q+1}^{M,N+2}\left[z\left|\begin{matrix}(1-a-s,\rho;1),(1-b,\sigma;1),A^{*}\\ B^{*},(1-a-b-s,\sigma+\rho;1)\end{matrix}\right.\right]+\sum_{t=0}^{\infty} y^{t+1} \overline{H}_{P+2,Q+1}^{M,N+2}\left[z\left|\begin{matrix}(1-a,\rho;1),(1-b-t,\sigma;1),A^{*}\\ B^{*},(1-a-b-t,\sigma+\rho;1)\end{matrix}\right.\right] \quad (2.1)$$

The formula (2.1) is valid, if the following (sufficient) conditions are satisfied.

$(i)\ \rho,\sigma>0 \qquad (ii)\ \Omega-\rho-\sigma>0\ ,|\arg z|<\frac{1}{2}(\Omega-\rho-\sigma)\pi$

$(iii)\max\{|x|,|y|\}<1 \quad or\ x=y=1 \ \ with \ \ Re(a)>1, \mathrm{Re}(b)>1$

Second formula

$$\sum_{m,n=0}^{\infty}\frac{x^{m}y^{n}}{m!n!}\overline{H}_{P+2,Q+1}^{M,N+2}\left[z\left|\begin{matrix}(1-a-m-n,u;1),(1-b-m,v;1),A^{*}\\ B^{*},(1-c-m,\omega;1)\end{matrix}\right.\right]$$

$$=\sum_{k=0}^{\infty}\frac{1}{k!}(1-y)^{-a}\left(\frac{x}{1-y}\right)^{k}\overline{H}_{P+2,Q+1}^{M,N+2}\left[z(1-y)^{-u}\left|\begin{matrix}(1-a-k,u;1),(1-b-k,v;)1,A^{*}\\ B^{*},(1-c-k,\omega;1)\end{matrix}\right.\right] \quad (2.2)$$

Provided that

$(i)u,v,\omega>0 \qquad (ii)\ \Omega-\omega>0,|\arg z|<\frac{1}{2}(\Omega-\omega)\pi$

$(iii)\ |x|+|y|<1\ and\ either\ \left|\frac{x}{1-y}\right|<1\ or\left|\frac{x}{1-y}\right|=1\ \ with\ \mathrm{Re}(c-a-b)>0$

Third formula

$$\sum_{m,n=0}^{\infty}\frac{x^{m}y^{n}}{m!n!}\overline{H}_{P+3,Q+2}^{M,N+3}\left[z\left|\begin{matrix}(1-a-m-n,u;1),(1-b-m,v;1),(1-b'-n,\omega;1)A^{*}\\ B^{*},(1-a-m,u;1),(1-a-n,u;1)\end{matrix}\right.\right]$$

$$=\sum_{k=0}^{\infty}\frac{1}{k!}(1-x)^{-b}(1-y)^{b'}\left(\frac{xy}{(1-x)(1-y)}\right)^{k}\overline{H}_{P+2,Q+1}^{M,N+2}\left[z(1-x)^{-v}(1-y)^{-u}\left|\begin{matrix}(1-b-k,v;1),(1-b'-k,\omega;1),A^{*}\\ B^{*},(1-a-k,u;1)\end{matrix}\right.\right] \quad (2.3)$$

Provided that

$(i)u,v,\omega>0 \qquad (ii)\ \Omega-2u>0,|\arg z|<\frac{1}{2}(\Omega-2u)\pi$

$(iii)\ |x|+|y|<1\ and\ either\ \left|\frac{xy}{(1-x)(1-y)}\right|<1\ or\left|\frac{xy}{(1-x)(1-y)}\right|=1\ \ with\ \mathrm{Re}(a-b-b')>0$

Fourth formula

$$\sum_{m,n=0}^{\infty}\frac{x^{m}y^{n}}{m!n!}\overline{H}_{P+3,Q+1}^{M,N+3}\left[z\left|\begin{matrix}(1-a-m-n,u;1),(1-b-m,v;1),(1-b'-n,\omega;1)A^{*}\\ B^{*},(1-b-b'-m-n,\omega+v;1)\end{matrix}\right.\right]$$

$$=\sum_{k=0}^{\infty}(1-y)^{-a}\frac{1}{K!}\left(\frac{x-y}{1-y}\right)^{k}\overline{H}_{P+3,Q+1}^{M,N+3}\left[z(1-y)^{-u}\left|\begin{matrix}(1-a-k,u;1),(1-b-k,v;1),(1-b',\omega;1),A^{*}\\ B^{*},(1-b-b'-k,\omega+v;1)\end{matrix}\right.\right]$$

(2.4)

Provided that

$(i)\, u,v,\omega>0 \qquad (ii)\ \Omega-v-\omega>0, |\arg z|<\frac{1}{2}(\Omega-v-\omega)\pi$

$(iii)\ \max\{|x|,|y|\}<1$, *either* $\left|\frac{x-y}{1-y}\right|<1$ *or* $\left|\frac{x-y}{1-y}\right|=1$ *with* $\operatorname{Re}(b'-a)>0$

In all the aforementioned formulas Ω is given by (1.3).

Derivation of the first formula: Using Mellin-Barnes type of contour integral (1.1) for the $\overline{H}$ -function occurring on the L.H.S. of (2.1) and changing the order of integration and summation, we find that L.H.S. of (2.1).

$$=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\overline{\phi}(\xi)z^{\xi}\frac{\Gamma(a+\rho\xi)\Gamma(b+\sigma\xi)}{\Gamma(a+b+(\sigma+\rho)\xi)}F_2\left[a+\rho\xi,b+\sigma\xi,1,1,;a+b+(\sigma+\rho)\xi;x,y\right]d\xi \tag{2.5}$$

Now appealing to a known result due to Srivastava ([8], p.1259, eq. (2.2))

$$F_2\left[a,b,1,1;a+b;x,y\right]=(x+y-xy)^{-1}\left\{x,\ {}_2F_1\left[a,1;a+b;x\right]+y\ {}_2F_1\left[b,1;a+b;x\right]\right\} \tag{2.6}$$

in (2.6) , we get L.H.S. of (2.1)

$$=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\overline{\phi}(\xi)z^{\xi}\frac{\Gamma(a+\rho\xi)\Gamma(b+\sigma\xi)}{\Gamma(a+b+(\sigma+\rho)\xi)}(x+y-xy)^{-1}$$

$$\left\{x\,{}_2F_1\left[a+\rho\xi,1;a+b+(\rho+\sigma)\xi;x\right]+y\,{}_2F_1\left[b+\sigma\xi,1;a+b+(\rho+\sigma)\xi;y\right]\right\}d\xi \tag{2.7}$$

Now expressing the ${}_2F_1$ functions in terms of their series and changing the order of integration and summation, and interpreting the result so obtained with the help of (1.1), we arrived at the formula (2.1).
Derivation of the formulas (2.2) to (2.4) : The summation formulas (2.2), (2.3) and (2.4) can be developed on the lines similar to the formula (2.1) except that, in place of(2.6) , here we use the following known results ([2],p.238,eq.(2),eq.(3) and eq.(1) respectively):

$$F_2\left[\alpha,\beta,\beta';\gamma,\beta;x,y\right]=(1-y)^{-\alpha}\ {}_2F_1\left[\alpha,\beta;\gamma;\frac{x}{(1-y)}\right] \tag{2.8}$$

$$F_2\left[\alpha,\beta,\beta';\alpha,\alpha;x,y\right]=(1-x)^{-\beta}(1-y)^{-\beta}\ {}_2F_1\left[\beta,\beta';\alpha;\frac{xy}{(1-x)(1-y)}\right] \tag{2.9}$$

$$F_2\left[\alpha,\beta,\beta';\beta+\beta';x,y\right]=(1-y)^{-\alpha}\ {}_2F_1\left[\alpha,\beta;\beta+\beta';\frac{x-y}{(1-y)}\right] \tag{2.10}$$

3. Summation Formulas:

If we take $x = y = 1$ in (2.1) and use the well known Gauss's summation theorem, we arrived at the result

$$\sum_{m,n=0}^{\infty}x^m y^n\overline{H}_{P+2,Q+1}^{M,N+2}\left[z\left|\begin{matrix}(1-a-m,\rho;1),(1-b-n,\sigma;1),A^{*}\\ B^{*},(1-a-b-m-n,\sigma+\rho;1)\end{matrix}\right.\right]=$$

$$\overline{H}_{P+2,Q+1}^{M,N+2}\left[z\left|\begin{matrix}(1-a-m,\rho;1),(2-b,\sigma;1),A^{*}\\ B^{*},(2-a-b,\sigma+\rho;1)\end{matrix}\right.\right]+\overline{H}_{P+2,Q+1}^{M,N+2}\left[z\left|\begin{matrix}(2-a,\rho;1),(1-b,\sigma;1),A^{*}\\ B^{*},(2-a-b,\sigma+\rho;1)\end{matrix}\right.\right] \tag{3.1}$$

Valid under the conditions of (2.1) .

Again if we put $x = y = \frac{1}{2}$ in (2.2), y = 1-x in (2.3) and x = 1 in (2.4) and make use of well known Gauss's summation theorem therein, we shall arrive at the following results

$$\sum_{m,n=0}^{\infty} \frac{(\frac{1}{2})^{m+n}}{m!n!} \overline{H}^{M,N+2}_{P+2,Q+1}\left[z \left| \begin{matrix} (1-a-m-n,u;1),(1-b-m,v;1),A^* \\ B^*,(1-c-m,\omega;1) \end{matrix} \right. \right]$$

$$= 2^a \overline{H}^{M,N+3}_{P+3,Q+2}\left[2^u z \left| \begin{matrix} (1-a,u;1),(1-b,v;1),(1-c+a+b,\omega-u-v;1) A^* \\ B^*,(1-c+a,\omega-u;1),(1-c+b,\omega-v;1) \end{matrix} \right. \right] \quad (3.2)$$

Where $\omega - u - v > 0, \omega \neq u \ \ or \ \ \omega \neq v$

And valid under the conditions of (2.1)

$$\sum_{m,n=0}^{\infty} \frac{x^m (1-x)^n}{m!n!} \overline{H}^{M,N+3}_{P+3,Q+2}\left[z \left| \begin{matrix} (1-a-m-n,u;1),(1-b-m,v;1),(1-b'-n,\omega;1) A^* \\ B^*,(1-a-m,u;1)(1-a-n,u;1) \end{matrix} \right. \right]$$

$$= x^{-b'}(1-x)^{-b} \overline{H}^{M,N+3}_{P+3,Q+2}\left[z x^{-\omega}(1-x)^{-v} \left| \begin{matrix} (1-b,v;1),(1-b',\omega;1),(1-a+b+b',u-v;1),A^* \\ B^*,(1-a+b,u-v;1),(1-a-b',u-\omega;1) \end{matrix} \right. \right] \quad (3.3)$$

Where $u - v - \omega > 0, \ u \neq \omega, \ u \neq v$

And valid under the conditions of (2.3).

$$\sum_{m,n=0}^{\infty} \frac{y^n}{m!n!} \overline{H}^{M,N+3}_{P+3,Q+1}\left[z \left| \begin{matrix} (1-a-m-n,u;1),(1-b-m,v;1),(1-b'-n,\omega;1),A^* \\ B^*,(1-b-b'-m-n,\omega+v;1) \end{matrix} \right. \right]$$

$$= (1-y)^{-a} \overline{H}^{M,N+3}_{P+3,Q+1}\left[z(1-y)^{-u} \left| \begin{matrix} (1-a,u;1),(1-b,v;1),(1-b'+a,\omega-u;1),A^* \\ B^*,(1-b-b'+a,\omega-u+v;1) \end{matrix} \right. \right] \quad (3.4)$$

Where $v \neq \omega, v \neq u$ and valid under the conditions (2.4).

4. Special Cases:

(i) In (2.1), taking M=1, N=0=P, Q=2, b_1=0, b_2= - λ , β_1 =1, $\beta_2 = \nu$, the $\overline{H}$ function reduces to generalized Wright-Bessel function $\overline{J}_{\lambda}^{\nu,\mu}$ ([4],p.271,eq.(8)) and we get

$$\sum_{m,n=0}^{\infty} x^m y^n \frac{\Gamma(a+m+\rho\xi)\Gamma(b+n+\sigma\xi)}{\Gamma(a+b+m+n+(\sigma+\rho)\xi)} \overline{J}_{\lambda}^{\nu,\mu}[z] = (x+y-xy)^{-1}$$

$$\left\{ \sum_{s=0}^{\infty} x^{s+1} \overline{H}^{1,2}_{2,3}\left[z \left| \begin{matrix} (1-a-s,\rho;1),(1-b,\sigma;1) \\ (0,1),(-\lambda,\nu;\mu),(1-a-b-s,\sigma+\rho;1) \end{matrix} \right. \right] + \sum_{t=0}^{\infty} y^{t+1} \overline{H}^{1,2}_{2,2}\left[z \left| \begin{matrix} (1-a,\rho;1),(1-b-t,\sigma;1) \\ (0,1),(-\lambda,\nu;\mu),(1-a-b-t,\sigma+\rho;1) \end{matrix} \right. \right] \right\} \quad (4.1)$$

where $(1-\nu) > 0, (1+\nu) \geq 0, |\arg z| < \frac{1}{2}(1-\nu-\rho-\sigma)\pi$ and the conditions (i) and (iii) given with (2.1) also satisfied.

(ii) In (2.1) replacing M,N,P,Q, by 1,P,P,Q+1 respectively, the $\overline{H}$ function reduces to the Wright's generalized hyper geometric function ${}_P\overline{\Psi}_Q([4], p.271, eq.(7))$ and we get

$$\sum_{m,n=0}^{\infty} x^m y^n \frac{\Gamma(a+m+\rho\xi)\Gamma(b+n+\sigma\xi)}{\Gamma(a+b+m+n+(\sigma+\rho)\xi)} {}_P\overline{\Psi}_Q\left[z \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,P} \\ (b_j,\beta_j;B_j)_{1,Q} \end{matrix} \right. \right]$$

$$= (x+y-xy)^{-1}\left\{\begin{array}{l}\sum_{s=0}^{\infty} x^{s+1}\overline{H}^{1,p+2}_{p+2,q+2}\left[z\left|\begin{array}{l}(1-a-s,\rho;1),(1-b,\sigma;1)(a_j,\alpha_j;A_j)_{1,P}\\(0,1),(b_j,\beta_j;B_j)_{1,Q},(1-a-b-s,\sigma+\rho;1)\end{array}\right.\right]+\\ \sum_{t=0}^{\infty} y^{t+1}\overline{H}^{1,p+2}_{p+2,q+2}\left[z\left|\begin{array}{l}(1-a,\rho;1),(1-b-t,\sigma;1),(a_j,\alpha_j;A_j)_{1,P}\\(0,1),(b_j,\beta_j;B_j)_{1,Q},(1-a-b-t,\sigma+\rho;1)\end{array}\right.\right]\end{array}\right\} \quad (4.2)$$

where

$\sum_{j=1}^{P}\alpha_j+1-\sum_{j=1}^{Q}\beta_j \equiv T>0;\quad |\arg z|<\frac{1}{2}(T-\rho-\sigma)\pi,\ 1+\sum_{j=1}^{Q}\beta_j-\sum_{j=1}^{P}\alpha_j \geq 0$ and the conditions (i) and (iii) given with (2.1) also satisfied.

(iii) The function $g_1=(-1)^p g(\gamma,\eta,\tau,p,z)\,([4],p.271,eq.(10))$ where

$$g_1=(-1)^p g(\gamma,\eta,\tau,p,z)=\frac{K_{d-1}\Gamma(p+1)\Gamma\left(\frac{1}{2}+\frac{\tau}{2}\right)}{2^{2+p}\pi^{1/2}\Gamma\left(\gamma-\frac{\tau}{2}\right)\Gamma(\gamma)}\overline{H}^{1,3}_{3,3}\left[-z\left|\begin{array}{l}(1-\gamma,1;1),(1-\gamma+\tau/2,1;1),(1-\eta,1;1+p)\\(0,1),(-\frac{\tau}{2},1;1),(-\eta,1;1+p)\end{array}\right.\right]$$

Where $K_d=\dfrac{2^{1-d}\pi^{-d/2}}{\Gamma(d/2)}$ $\quad([6],p.4121,eq.(5))$ (4.3)

From this we get

$$\sum_{n,n=0}^{\infty} x^m y^n \frac{K_{d-1}\Gamma(p+1)\Gamma\left(\frac{1}{2}+\frac{\tau}{2}\right)\Gamma(a+m+\rho\xi)\Gamma(b+n+\sigma\xi)}{2^{2+p}\pi^{1/2}\Gamma\left(\gamma-\frac{\tau}{2}\right)\Gamma(\gamma)\Gamma(a+b+m+n+(\rho+\sigma)\xi)}$$

$$\overline{H}^{1,3}_{3,3}\left[z\left|\begin{array}{l}(1-\gamma,1;1),(1-\gamma+\tau/2,1;1),(1-\eta,1;1+p),(1-a-m,\rho;1),(1-b-n,\sigma;1)\\(0,1),(-\frac{\tau}{2},1;1),(-\eta,1;1+p),(1-a-b-m-n,\sigma+\rho;1)\end{array}\right.\right]$$

$$=\left(x+y-xy\right)^{-1}\frac{K_{d-1}\Gamma(p+1)\Gamma\left(\frac{1}{2}+\frac{\tau}{2}\right)}{2^{2+p}\pi^{1/2}\Gamma\left(\gamma-\frac{\tau}{2}\right)\Gamma(\gamma)}$$

$$\{\sum_{s=0}^{\infty} x^{s+1}\overline{H}^{1,5}_{5,4}\left[z\left|\begin{array}{l}(1-\gamma,1;1),(1-\gamma+\tau/2,1;1),(1-\eta,1;1+p),(1-a-s,\rho;1),(1-b,\sigma;1)\\(0,1),(-\frac{\tau}{2},1;1),(-\eta,1;1+p),(1-a-b-s,\sigma+\rho;1)\end{array}\right.\right]+$$

$$\sum_{t=0}^{\infty} y^{t+1}\overline{H}^{1,5}_{5,4}\left[z\left|\begin{array}{l}(1-\gamma,1;1),(1-\gamma+\tau/2,1;1),(1-\eta,1;1+p),(1-a,\rho;1),(1-b-t,\sigma;1)\\(0,1),(-\frac{\tau}{2},1;1),(-\eta,1;1+p),(1-a-b-t,\sigma+\rho;1)\end{array}\right.\right]\} \quad (4.4)$$

valid under the conditions of (2.1).

Further if we take $\rho\to 0$ in (2.1), we get the following new transformation formula:

$$\sum_{m,n=0}^{\infty} x^m y^n (a)_m \overline{H}^{M,N+1}_{P+1,Q+1}\left[z\left|\begin{array}{l}(1-b-n,\sigma;1),A^*\\B^*,(1-a-b-m-n,\sigma+\rho;1)\end{array}\right.\right]$$

$$= (x+y-xy)^{-1}\left\{\sum_{s=0}^{\infty} x^{s+1}(a)_s \overline{H}_{P+1,Q+1}^{M,N+1}\left[z\left|\begin{matrix}(1-b,\sigma;1),A^*\\ B^*,(1-a-b-s,\sigma;1)\end{matrix}\right.\right]+\sum_{t=0}^{\infty} y^{t+1}\overline{H}_{P+1,Q+1}^{M,N+1}\left[z\left|\begin{matrix}(1-b-t,\sigma;1),A^*\\ B^*,(1-a-b-t,\sigma;1)\end{matrix}\right.\right]\right\} \quad (4.5)$$

Valid under the conditions of (2.1) .
A similar type of result can be obtained by taking $\sigma \to 0$ in (2.1).

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